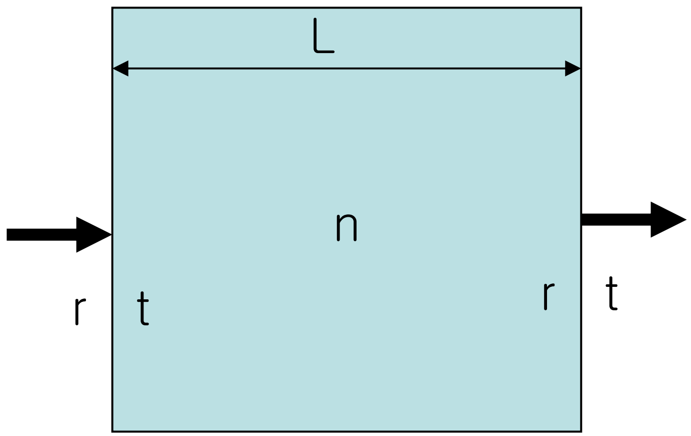
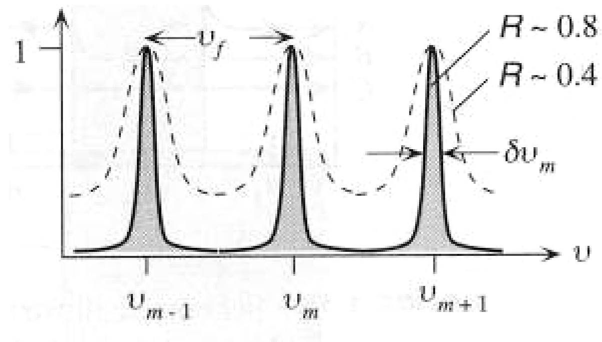
Lect. 6: Interferometers

Fabry-Perot Interferometer





$$T = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(kL)}$$

In general, two parallel partially reflecting mirrors

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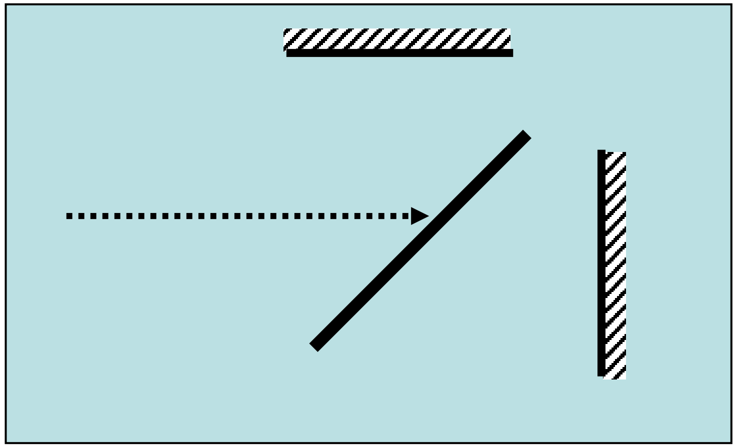
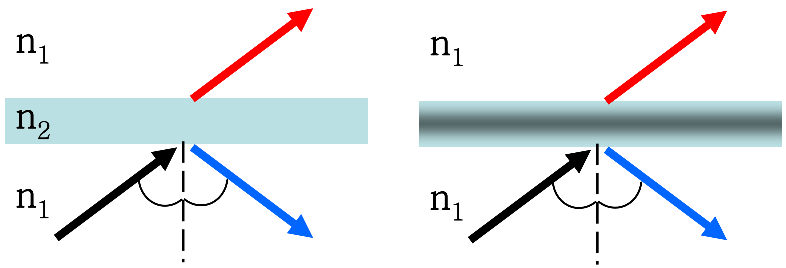
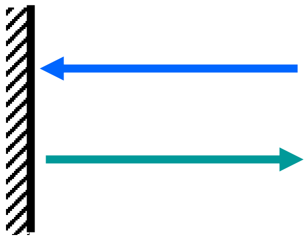
Michelson Interferometer: Two mirrors and one beam splitter



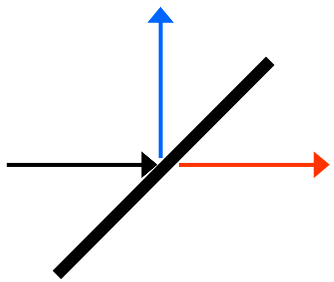


- Mirror (perfect conductor)



R=1, r=−1

- Beam Splitter



$$R = \frac{1}{2},\ T = \frac{1}{2}$$

$$r = ?,\ t = ?$$

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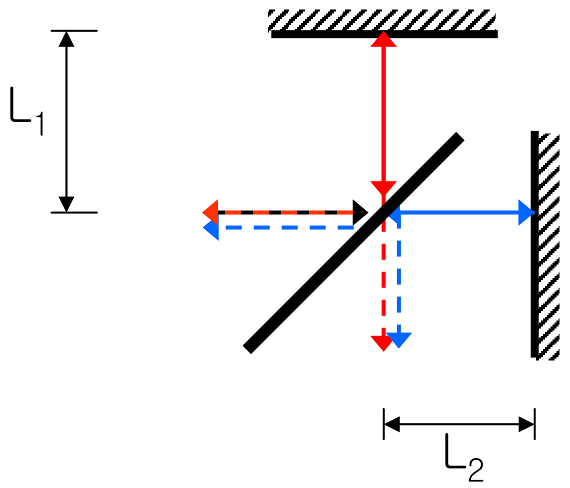
Michelson Interferometer:



Four outputs:

Side 1: r r
2: t t

Bottom 1: r t
2: t r

$$E_{out,\,side} = re^{-jkl_1}(-1)e^{-jkl_1}r + te^{-jkl_2}(-1)e^{-jkl_2}t = -r^2e^{-j2kl_1} - t^2e^{-j2kl_2}$$

$$E_{out,\,bottom} = re^{-jkl_1}(-1)e^{-jkl_1}t + te^{-jkl_2}(-1)e^{-jkl_2}r = -rte^{-j2kl_1} - rte^{-j2kl_2}$$

With $r = t = \frac{1}{\sqrt{2}}$,

$$E_{out,\,side} = -r^2 e^{-j2kl_1} - t^2 e^{-j2kl_2}$$

$$= -\frac{1}{2}\left(e^{-j2kl_1} + e^{-j2kl_2}\right) = -\frac{1}{2} e^{-jk(l_1+l_2)}\left(\mathrm{e}^{-jk(l_1-l_2)} + \mathrm{e}^{jk(l_1-l_2)}\right)$$

$$I_{out,\,side} = \left|E_{out,\,side}\right|^2 = \cos^2[k(l_1 - l_2)]$$

$$E_{out,\,bottom} = -rte^{-j2kl_1} - rte^{-j2kl_2}$$

$$= -\frac{1}{2}\left(e^{-j2kl_1} + e^{-j2kl_2}\right)$$

$$I_{out,\,bottom} = \left|E_{out,\,bottom}\right|^2 = \cos^2[k(l_1 - l_2)]$$

Against energy conservation!

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With $r = \frac{1}{\sqrt{2}},\ t = j\frac{1}{\sqrt{2}}$

(r and t should have $\pi/2$ phase difference in order to satisfy energy conservation)

$$E_{out,\,side} = -r^2 e^{-j2kl_1} - t^2 e^{-j2kl_2}$$

$$= -\frac{1}{2}\left(e^{-j2kl_1} - e^{-j2kl_2}\right) = -\frac{1}{2} e^{-jk(l_1+l_2)}\left(\mathrm{e}^{-jk(l_1-l_2)} - \mathrm{e}^{jk(l_1-l_2)}\right)$$

$$I_{out,\,side} = \left|E_{out,\,side}\right|^2 = \sin^2[k(l_1 - l_2)]$$

$$E_{out,\,bottom} = -rte^{-j2kl_1} - rte^{-j2kl_2}$$

$$= -\frac{j}{2}\left(e^{-j2kl_1} + e^{-j2kl_2}\right)$$

$$I_{out,\,bottom} = \left|E_{out,\,bottom}\right|^2 = \cos^2[k(l_1 - l_2)]$$

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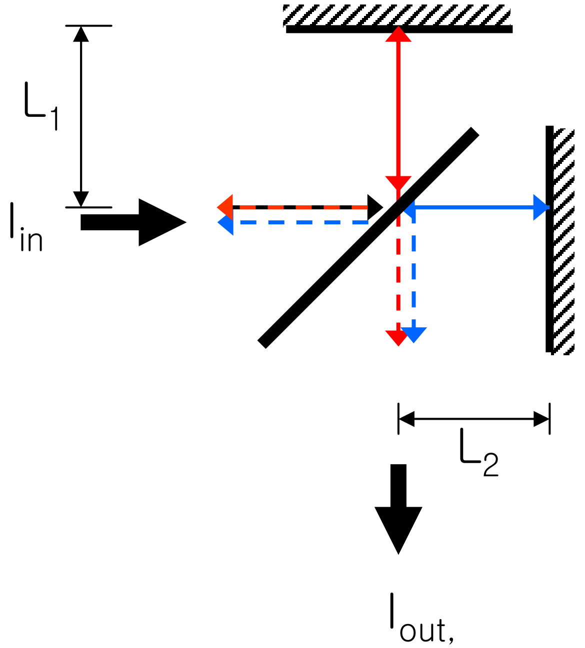
Michelson Interferometer:



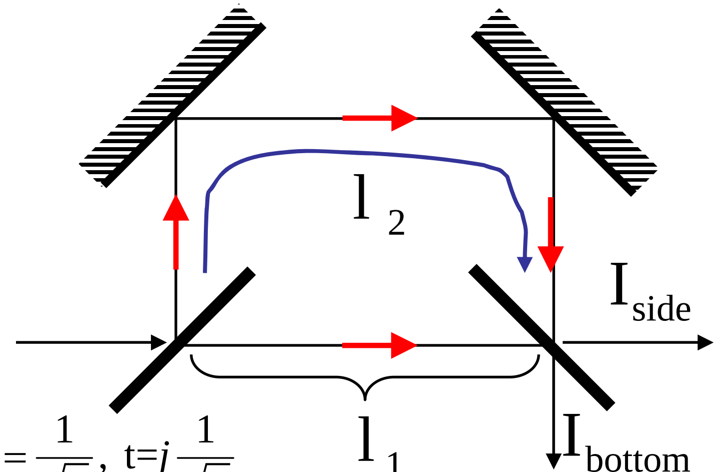
$$\frac{I_{out,bottom}}{I_{in}} = \cos^2[k(l_1 - l_2)];$$

$k(l_1 - l_2)$ can be measured very precisely

- Experimental measurement of speed of light by Michelson in 1879.

- Provided a clue for Special Relativity– speed of light is same always; Michelson Morley experiment

Mach-Zehnder Interferometer:



$r = \frac{1}{\sqrt{2}}$, t=$j\frac{1}{\sqrt{2}}$

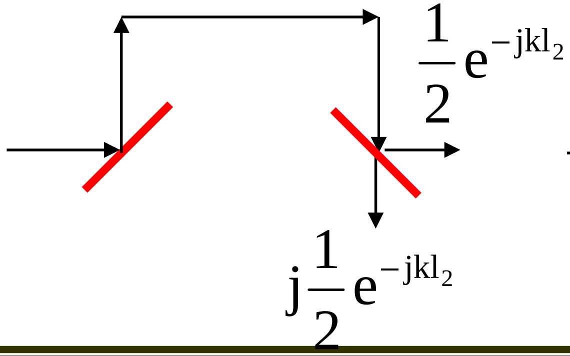
$$E_{out,\,side} = \frac{1}{2}\left(e^{-jkl_2} - e^{-j2kl_1}\right) = \frac{1}{2}e^{-jk\frac{l_2+l_1}{2}}\left(e^{-jk\frac{l_2-l_1}{2}} - e^{jk\frac{l_2-l_1}{2}}\right)$$

$$I_{out,\,side} = \sin^2\left(k\frac{l_1-l_2}{2}\right)$$

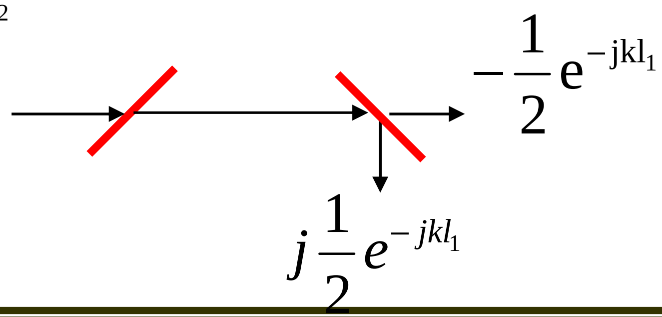
$$E_{out,\,bottom} = \frac{j}{2}\left(e^{-jkl_1} + e^{-jkl_2}\right) = \frac{j}{2}e^{-jk\frac{l_1+l_2}{2}}\left(e^{-jk\frac{l_1-l_2}{2}} + e^{jk\frac{l_1-l_2}{2}}\right)$$

$$I_{out,\,bottom} = \cos^2\left(k\frac{l_1-l_2}{2}\right)$$

Case#1



Case#2



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Can any EM wave cause interference?
Only EM waves having clear phase relationship cause interference: coherent

How large (l_1−l_2) can be?
Any long as two separated waves are coherent or within coherent length

Separated waves become incoherent due to intrinsic phase noises

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Exercise Problems:

Prob. 2, 4, 10